COLUMBIA BUSINESS SCHOOL

Decision Models

Decision modeling refers to the use of mathematical or scientific methods to determine an allocation of scarce resources which improves or optimizes the performance of a system.

What is Decision Modeling?

The terms *operations research* and *management science* are also used to refer to decision modeling.

Lecture 1

- Syllabus Overview
- Introduction to Decision Models
- Bland Brewery Linear Programming Model
- Spreadsheet Optimization
- Summary and Preparation for next class

 Real World System
 Formulation
 Decision Model

 Implementation
 Deduction

 Real World Conclusions
 Interpretation

Decision Modeling Process

Applications of Decision Models

A sample of *systems* to which decision models have been applied include:

- Financial systems
 - Portfolio optimization, security pricing (e.g., options, mortgage-backed securities), cash flow matching (e.g., pension planning and bond refunding)
 - Example: LibertyView Capital Management uses a spreadsheet optimization model developed by a 1995 Columbia MBA to hedge bond investments using stock and options

• Production systems

- ▷ oil, steel, chemical, and many other industries
- Example: Citgo uses linear programming to improve refining operations. Total benefit: approximately \$70 million annually.

Applications of Decision Models (continued)

- Distribution systems
 - ▷ airlines, paper, school systems, and others
 - Example: Westvaco, a Fortune 200 paper company, uses linear programming to optimize its selection of motor carriers. The result: 3–6% savings on trucking costs of \$15 million annually. This work was done by a 1992 Columbia MBA.
- Marketing systems
 - sales force design, forecasting new product sales, telecommunications strategies, brand choice, merchandising strategies
- Graduate school admissions
 - Example: The director of CBS admissions uses linear programming to aid in the admissions process.

References: The journal *Interfaces*, and the book *Excellence in Management Science Practice*, by Assad, Wasil, and Lilien, Prentice Hall, Englewood Cliffs, NJ (both are in the business school library).

Overview of Decision Models



Bland Brewery Decision Problem

Consider the situation of a small brewery whose ale and beer are always in demand but whose production is limited by certain raw materials that are in short supply. The scarce ingredients are corn, hops, and barley malt. The recipe for a barrel of ale calls for the ingredients in proportions different from those in the recipe for a barrel of beer. For instance, ale requires more malt per barrel than beer does. Furthermore, the brewer sells ale at a profit of \$13 per barrel and beer at a profit of \$23 per barrel. Subject to these conditions, how can the brewery maximize profit?

Main solution tools

- Optimization
 - Linear programming, Integer programming, Nonlinear programming
- Simulation



Bland Brewery Model

What if Bland decides to produce all ale? Then

- $\circ~$ their corn supply limits production to at most 480/5 = 96 barrels,
- their hops supply limits production to at most 160/4 = 40 barrels, and
- their malt supply limits production to at most 1190/35 = 34 barrels.

Therefore, they can produce only 34 barrels of ale, which makes a profit of $34 \times \$13 = \442 .

What if Bland decides to produce all beer? Then

- their corn supply limits production to at most 480/15 = 32 barrels,
- their hops supply limits production to at most 160/4 = 40 barrels, and
- their malt supply limits production to at most 1190/20 = 59.5 barrels.

Therefore, they can produce only 32 barrels of beer, which makes a profit of $32 \times \$23 = \736 .

Is there a better production plan? One way to simplify the computations is to set up a spreadsheet.

Cell F5: =SUMPRODUCT(C4:D4,C5:D5)

						2 S 2 S						
A B	C	D	E	E F	G	н		10	A	8	C	D
1 BLAND XLS	Bland Breve	iny Produc	Siot Proble	m			10	1	BLAND X	s	Etand Brown	Wy Prod
2 1 Product	Ale	Plant		Total	>			2	Thursday			Rea.
Elarrels to Produce	5	10		Profit				1	Barrels to	Produce	4	10
5 Profit per Barrel	\$13	\$23	1		1			5	Profit per	Daret	\$13	\$23
8								. 6	10000			
7	Product Ingr	edient Res	quirements	11	-			7			Producting	echent R
			Total		Guantity			10	a second	2 C - 1		Base
S ingredients	100	Clear	Usage		AV8 SDIA			1	Condes		5	10
44 Line (etc)	1	10			350			1	Hops (ogs	ő	4	
12 Mat (bc)	36	20			1190			10	Mult (kni)	S	35	20
13	100	-			1100			1	8			
14								1	8			
15							-		el el el altres d	(New 7 7 1	AND / Chant /	Canal St
HITER, Bard / Stand J St	hand 2 of Shouth of	Sheets 7, Sh	00523111				10		ada		week anney	1
Ready		Ser-0.			the Real Property lies		100					1

Figure 1. The preliminary spreadsheet BLAND.XLS



Figure 2. The spreadsheet BLAND.XLS with formulas

X Alternal Local - Manif				- 171-
Diffe En Yees boat i	flenn Joop Day He	dow Çal Ayı Hel		_101×
Lan 11	· · · · · ·		· 34 /3 11.	21- 21-
nisial albieta	inter to late	wind entry a	dial laboration in the	- Olari
	al minimized as in the	cal, loairei c	line her he	
		ACT IN CALLED IN		
AL 2	BLAND XLS			
A Street	a amotors		12	H a
T DUNUAL INTER	and I		Contract of the	
1 Contest Aurily	AND THE FRE		100	
d Flavals to 5 Bullio	the second second	-	Base	
Codd tour R	de firme	ner l	1	
d Color	to the Tanan Section S		manual distances of the	
2	NOT BELOW BELOW DE	- I - I - I	Chenne 1	
0		- Andrew -	1 MIN	
g Instactant		Direct	Special Star	
10 Corr (bs)		-1 Drive	34 10	
11 Hops (025)		- Andrewski -	03	
12 Mat (bs)	35 2	0 375	1190	
13				
14				
15				
#[#[#[#]#####ZSteel.23	Aut / Short / Short /	Sheet5 2 3 1 + 1		1 11
Erner	Sec.	0		

Figure 3. The Solver Parameters dialog box

A description of the Excel spreadsheet optimizer is given in the reading "An Introduction to Spreadsheet Optimization using Excel."

ol		12121		00 2	14 91 91		1175		10
		-		ada-1 S	Share Ball		H	2.02	3
_	15 m	5	ANDINES	_	_		-		_
12	A	Contract of the local division of the local	-		-			н	
1	BLAND X			_		1			
2		Pill Linder C	of Ran		-	Seke.	1		
3	Froduct	Equato -	Her C.H.	to C.Maler	- P.	S Chief	1		
4	Elarrels to F	B Change	g Cells:	and the second second se	Contractory of the local division of the loc		1.1		
5	Profit per 8	10.01 (0.0	1		1 Arres	1 Courses	100		
đ		Space	Ne Constants;			Dataset	1		
7		161016	1214 (01101	54/2	- MA	1.11.1			
0		line.			-	1	181Y		
8	Ingredients				Number	1 100.0	the .		
0	Cort (bs)				-I Deter		100		
11	Hops (ozs)		57.57				50		
12	(Mat (lbs)		35	20	980	s=:	1190		
13									
14	1								

Figure 4. The *Solver Parameters* dialog box with constraints added

A B	6	0	E #	0	H	1 3
CLAR ALS	Calves Options		1	82		
Product	Au. 300 1	-	OK	1		
Carrels to Produce	postere.	-	Death			
Profit per Darreit	bases 5	-010K	maintent.			
	times. F		Sugar	2		
	Second F	And in case of the local division of the loc		Quette		
Renderts	Station .	100 200		S-minter		
Carri (ba)	C insure boots	and Car	an Developer Property	480		
Hape (cza)	Citral of	Developed	Seech .	1.00		
CHED MARK	(a particular	C. Darmer	N Queres	1190		
	(Janes	of the states	- oppose			

Figure 5. The *Solver Options* dialog box

Differ Eak Years journed Far	and Jook D	at the set	OF BA S	946 - 1 - 1	eal and result	and a	iel-
				56 + 1		21-1	-
	101 101	*7 C# 1	Tel SHITH	213	and huse	2.2	100
		44 11- 5		田丁			
AL 20 5	LANDXLS						-
A B	C	D	E	P	G	H	1
1 840035	Bland Breve	iny Produc	Sot Problem	9			
2	1000	1000		201			
3 Product	Ale	Dear		Total			
Barrels to Produce	.12	20	-	Profit	-		
6 Profit per Barrel	\$13	\$23	L	\$800			
6							
7	Product Ingr	ed ent Rec	purements				
0			Total		Guentity		
9 Inspecients	Ne	Deer	Usage		Available		
10 Cort (be)	- 5	15	400		400		
11 Hops (025)	4	4	160		160		- 8
12 Mat (bs)	35	20	980		1180		
13							
14							
19							- 2
interfactory, preval 2 Street 2 Ste	er (steel (10005 X 584	H0 X 0 1 1			-	10
second.		The Arth					

Figure 6. The spreadsheet after optimizing





Bland Brewery LP - Standard Notation

Decision Variables:

Let A = # of barrels of ale to produce, and

B = # of barrels of beer to produce.

Note: Use suggestive (mnemonic) variable names for readability.

Bland Brewery Linear Program

max	13A + 23B	(Profit)
subject to:		
(Corn)	$5A+15B~\leq$	480
(Hops)	$4A + 4B \leq$	160
(Malt)	$35A+20B~\leq$	1190
(Nonnegativity)	$A, B \ge$	0

Terminology

Feasible and Infeasible Solutions

A production plan (A, B) which satisfies all of the constraints is called a *feasible solution*. For example, in the Bland Brewery LP, the solution (A = 10, B = 10) is feasible. The production plan (A = 40, B = 10) is not feasible, i.e., it is *infeasible* because the hops and malt constraints are *violated*.

Optimal Solution

For a maximization (respectively, minimization) problem, an *optimal solution* is a feasible solution that has the largest (respectively, smallest) objective function value among all feasible solutions.

The optimal solution for the Bland Brewery production model is (A = 12, B = 28). This means that Bland's optimal production plan is to produce 12 barrels of ale and 28 barrels of beer. The optimal objective function value is \$800.

Assumptions in a Linear Program

- Continuity: the decision variables are continuous, i.e., fractional values are allowed.
- Proportionality: for example, it takes twice as much hops to make twice as much beer or ale; there are no economies of scale.
- Additivity: profit is the *sum* of the profit contributions from ale and beer.

In short, the objective function and constraints must be *linear*. For example, 13A + 23B is a linear function of *A* and *B*. The functions $13A^2 + 23AB$ and $\log(A) + \cos(B)$ are *nonlinear functions*. The function max(*A*, 0) is *not differentiable* at *A* = 0 and =IF(*A* < 5,0,10) is a *discontinuous* function.

Allowable variations:

- Objective function can be *maximized* or *minimized*.
- $\circ~$ Constraints can be " \geq ," " \leq ," or "=."
- Noninteger or integer coefficients and righthand sides are allowed.
- Negative or positive coefficients and righthand sides are allowed.

Summary

- Understand LP terminology: decision variables, objective function, constraints, feasible and infeasible solutions, optimal solution
- Formulate simple linear programs
- Solve simple linear programs in a spreadsheet

Preparation for next class

- Formulate and solve the "Shelby Shelving" case (in the readings book or on pp.68-69 in the W&A text). Prepare to discuss the case in class, but do not write up a formal solution.
- Readings: "An Introduction to Spreadsheet Optimization Using Excel" in the readings book.
 Read Chapter 1 and Chapter 2.1–2.4, 2.6, 2.8 and Chapter 4.1, 4.2 in the W&A text.
- Optional readings: "OR Brews Success for San Miguel" and "Logistics Steps Onto Retail Battlefield" in the readings book.